Lecture 3 – Power Transformers

Core structures. Winding connections. Equivalent circuits.

Power Transformers

Core Structures of Three-Phase Transformers

A three-phase transformer can be considered to be some combination of single-phase transformers, either as three separate units, or as a single unit with three sets of phase windings on a common magnetic core.

A single unit construction permits some considerable saving of materials, and is therefore the usual option. Using three separate single-phase units is sometimes necessary for very large transformers to overcome weight and size limitations of transport. While the three separate single-phase units cost more than the equivalent three-phase unit, there is a saving in the cost of the spare transformer, usually mandatory for the security of supply.

The standard configurations of three-phase cores are:

(a) Three-Limb Core

This is the most common arrangement, and uses the least amount of core material. The parts of the core joining the three limbs are known as the yokes.

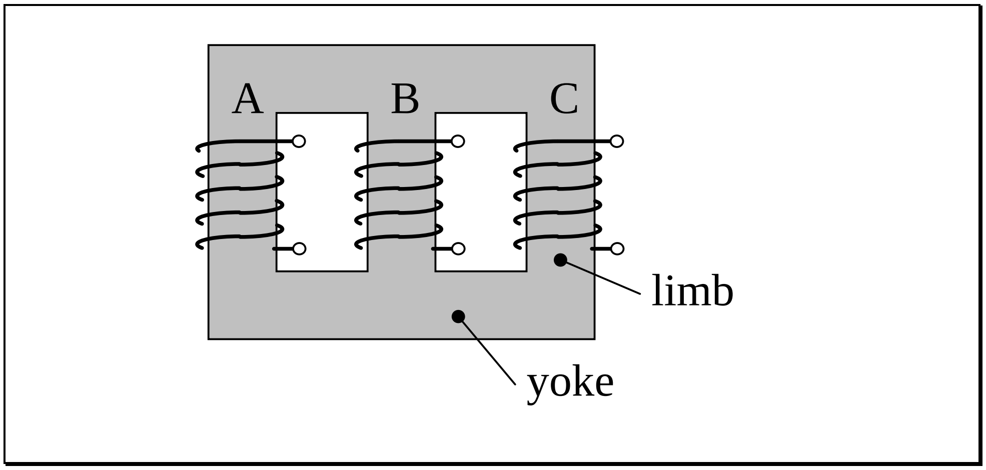


Figure 3.1 – Three-limb core

(b) Five-Limb Core

In the three-limb arrangement the yokes have the same cross-sectional area as the limbs. With two extra unwound limbs the top and bottom yokes can be reduced to half the cross-section of the three wound limbs. The unwound limbs also need to have only half the cross-section of the wound limbs, so the extra steel required for the unwound limbs comes mainly from the steel saved in the yokes.

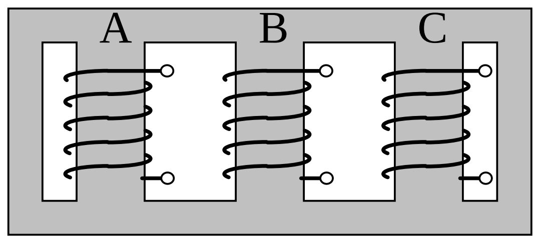


Figure 3.2 – Five-limb core

The lower profile yokes lead to a reduction of height. Therefore the five-limb core is commonly used in large transformers, where the transport height is an important design limitation.

Both of the arrangements (a) and (b) are known as *core type*, as opposed to the *shell type* below.

(c) Shell Type Core

This type uses almost as much core steel as three separate cores. It does have a design advantage of permitting winding construction with well supported coils, to provide mechanical strength to withstand short-circuit forces. The shell type arrangement is comparatively rare, and used only for very large transformers.

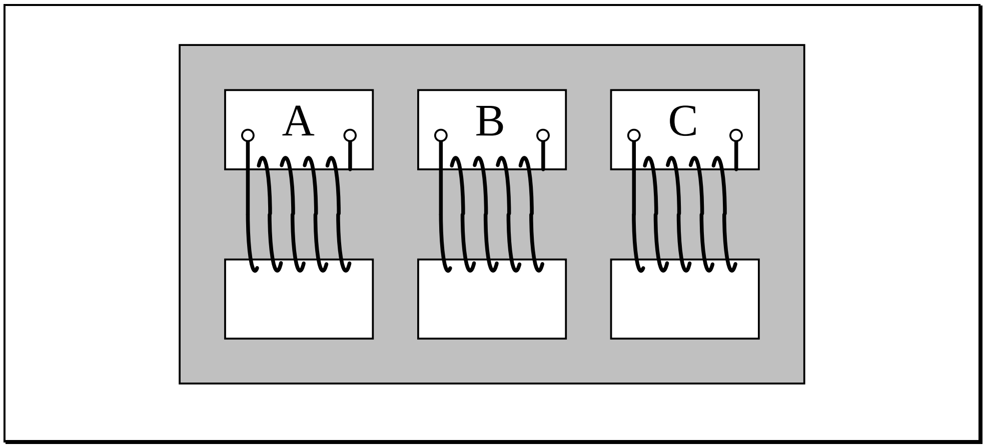


Figure 3.3 – Shell-type core

Winding Connections of Three-Phase Transformers

The basic connections are delta, star, and zig-zag (interconnected star). Any combination of these basic winding connections, with variations in polarities, may be found in a transformer.

The connections of a particular transformer are indicated by a *connection symbol*, sometimes called a *vector symbol*. The possible connections result in various inherent phase displacements between primary and secondary voltages. The standard phase displacements are 0°, 180°, -30° and +30°, but other values are possible (e.g. with the zig-zag connection, or with a delta autotransformer).

The connection symbol for a two-winding transformer is composed as follows:

HV symbol	HV neutral symbol	LV symbol	LV neutral symbol	Phase displacement symbol
"D" for delta "Y" for star "Z" for zig-zag	"N" if neutral terminal is accessible	"d" for delta "y" for star "z" for zig-zag	"n" if neutral terminal is accessible	"0" for 0° "6" for 180° "1' for -30° "11" for +30°

The phase displacement symbol is a clock hour figure showing the position of the equivalent star voltage phasor for the LV winding, with the corresponding HV phasor in the reference position of zero or 12 o'clock.

Examples

Yyn0 = star-star, HV neutral not available, LV neutral available, zero phase displacement.

YNzn1 = HV star, HV neutral available, LV zig-zag, LV neutral available, LV lags HV by 30°.

For multi-winding transformers the HV winding (the highest voltage) remains the reference for phase displacement, and its symbol is written first. Other symbols follow in diminishing order of rated voltages, and preferably separated with commas for clarity.

Example

A transformer has three windings:

- 132 kV star (HV) with neutral brought out
- 36 kV star, in phase with HV, with the neutral brought out
- 7.2 kV delta, leading the HV by 30°

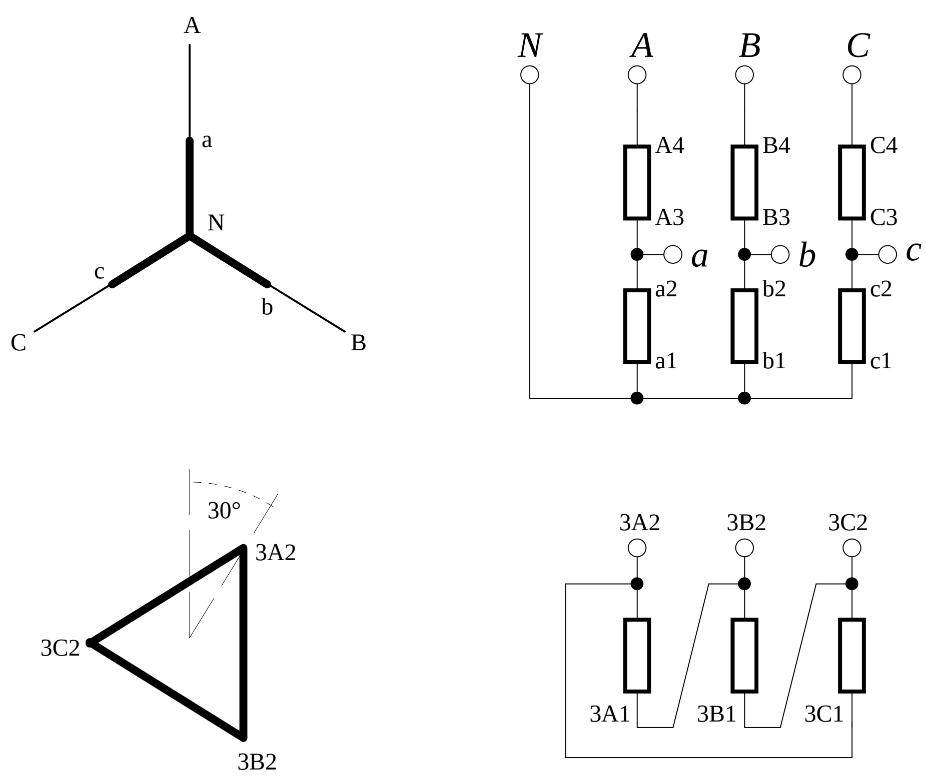
The connection symbol for this transformer is: YN, yn0, d11.

For autotransformers, in which the two windings have a common part, the letter “a” is used to designate the lower voltage of the auto-connected pair, and is placed immediately after the symbol for the higher voltage of the pair. Three-phase autotransformers are usually star connected.

Examples

D, yn11, a11 — A separate high voltage delta winding, and an intermediate to low voltage star autotransformer. The intermediate and low voltages lead the high voltage by 30°.

YN, a0, d1 — A HV auto-connected star winding with the neutral terminal brought out, and a separate delta winding. The equivalent star voltage of the delta winding lags the HV by 30°. Diagram of connections as follows:



Reference and Terminology

AS 2374 – Power Transformers, Part 4 – Tappings and Connections (1982) is the relevant Australian Standard that deals with transformer winding connections. In AS 2374 the term *winding*, for three-phase transformers, refers to the three *phase windings* associated with one of the voltages assigned to the transformer. Hence the terms *high voltage winding*, *low voltage winding*, and *intermediate voltage winding* for the main windings of a transformer. These terms, unlike *primary* and *secondary* do not imply any set direction of power flow.

It is customary to add a third delta-connected auxiliary winding to all large star-star transformers to decrease the zero sequence impedance. This third winding is generally known as the *tertiary winding*, although AS 2374 does not use the term. Generally the tertiary winding has a lower MVA rating than the main windings. The tertiary winding, if its terminals are brought out, can also be used to supply a small load.

Equivalent Circuits of Power Transformers

Single-Phase Transformers

(a) Ideal Transformers

Many power transformer problems can be solved with sufficient accuracy by treating the transformers as ideal:

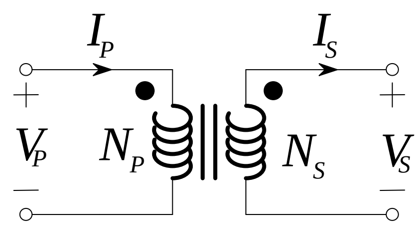


Figure 3.4 – Two-winding ideal transformer

For the two-winding ideal transformer:

$$\frac{V_P}{N_P} = \frac{V_S}{N_S} \tag{3.1}$$

$$N_P I_P = N_S I_S \tag{3.2}$$

3.8

The three-winding ideal transformer is represented by:

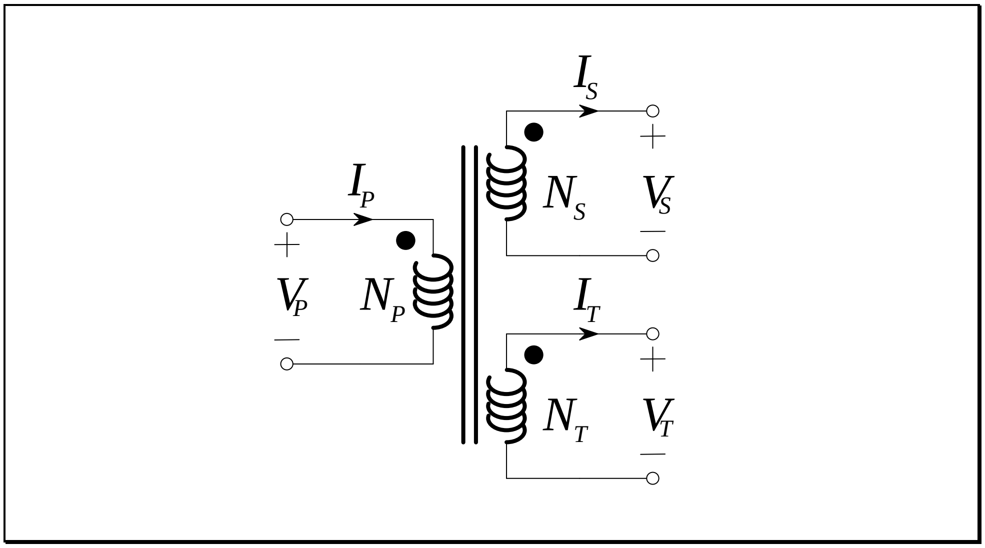


Figure 3.5 – Three-winding ideal transformer

and has ideal equations:

$$\frac{V_P}{N_P} = \frac{V_S}{N_S} = \frac{V_T}{N_T} \tag{3.3}$$

$$N_P I_P = N_S I_S + N_T I_T \tag{3.4}$$

Let:

$$a_{PS} = \frac{N_P}{N_S} = \text{primary to secondary turns ratio} \tag{3.5}$$

$$a_{PT} = \frac{N_P}{N_T} = \text{primary to tertiary turns ratio} \tag{3.6}$$

Then:

Secondary voltage referred to the primary $V_S' = a_{PS} V_S$ (3.7)

Tertiary voltage referred to the primary $V_T' = a_{PT} V_T$ (3.8)

Secondary current referred to the primary $I_S' = \frac{I_S}{a_{PS}}$ (3.9)

Tertiary current referred to the primary $I_T' = \frac{I_T}{a_{PT}}$ (3.10)

Hence, for the ideal transformer:

$$\boxed{V_P = V_S' = V_T'} \tag{3.11}$$

$$\boxed{I_P = I_S' + I_T'} \tag{3.12}$$

and the equivalent circuit, in terms of the referred values, is shown below:

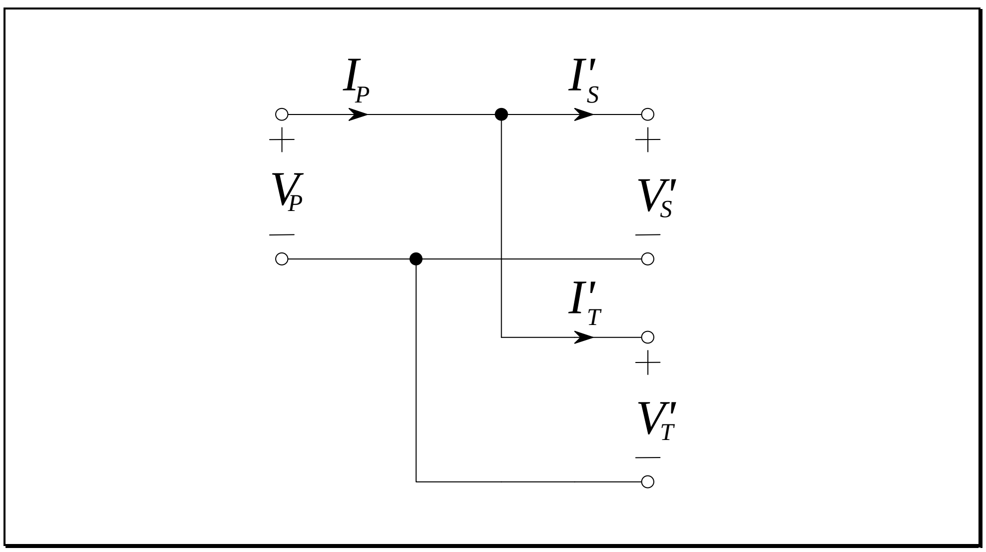


Figure 3.6 – Three-winding ideal transformer with referred values

(b) Practical Power Transformers

Practical transformers have significant leakage reactance X_L and resistance R_L. They also draw magnetising current, but for most power system calculations this can be ignored. Therefore we use the simplified equivalent circuits shown below for two- and three-winding transformers.

The equivalent circuit for a two-winding power transformer is:

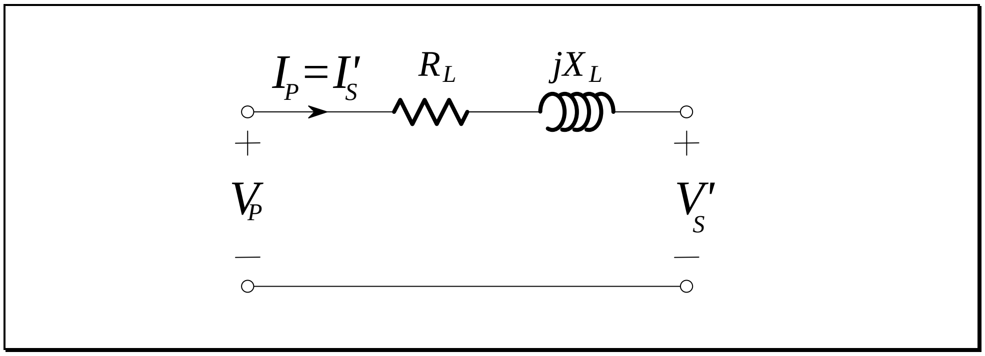


Figure 3.7 – Equivalent circuit of a two-winding power transformer

When using per-unit values the term “referred to the primary” is superfluous, and we can replace V'_S and I'_S with V_S and I_S respectively.

The equivalent circuit for a three-winding power transformer is:

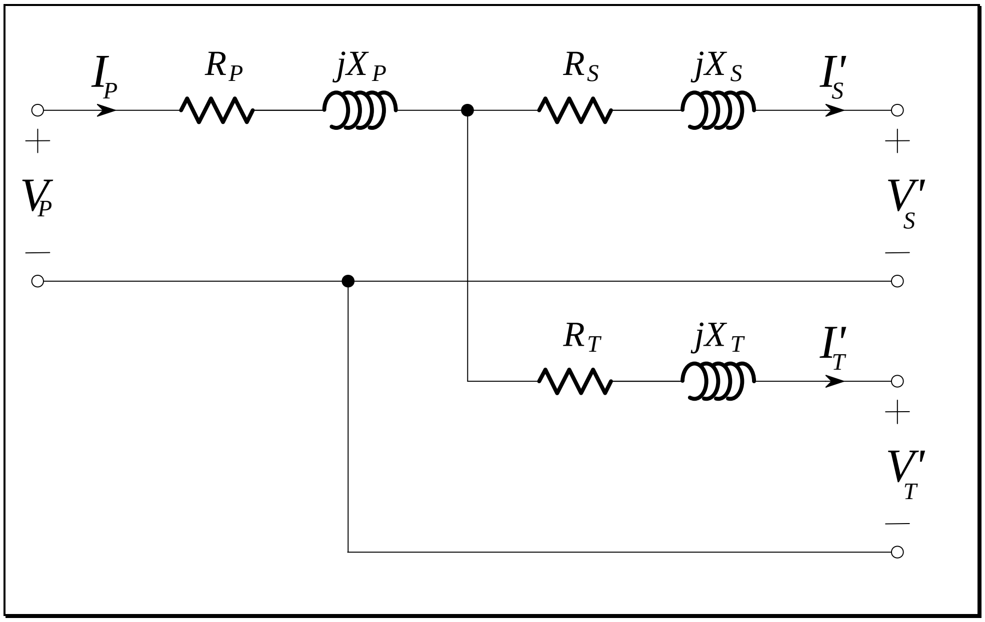


Figure 3.8 – Equivalent circuit of a three-winding power transformer

The equivalent circuit impedances:

$$
\begin{aligned}
Z_P &= R_P + jX_P \\
Z_S &= R_S + jX_S \\
Z_T &= R_T + jX_T
\end{aligned}
\tag{3.13}
$$

are not directly measurable, and have no physical meaning. They are merely abstract components of the equivalent circuit. The measurable impedances are:

$$
\begin{aligned}
Z_{PS} &= Z_P + Z_S \\
Z_{PT} &= Z_P + Z_T \\
Z_{ST} &= Z_S + Z_T
\end{aligned}
\tag{3.14}
$$

Solving the simultaneous equations (3.14) gives:

$$
\begin{aligned}
Z_P &= \frac{1}{2}\sum Z - Z_{ST} \\
Z_S &= \frac{1}{2}\sum Z - Z_{PT} \\
Z_T &= \frac{1}{2}\sum Z - Z_{PS}
\end{aligned}
\tag{3.15}
$$

where:

$$
\sum Z = Z_{PS} + Z_{PT} + Z_{ST} \tag{3.16}
$$

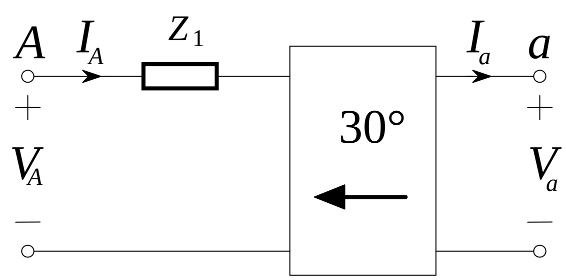
In the interest of generality we should note that transformer windings also have some effective capacitance. The capacitance is important in some types of transformer (e.g. high voltage testing transformers), but can be safely ignored in power transformers, as long as we are concerned only with steady-state performance.

Three-Phase Transformers – Positive Sequence Equivalent Circuits

For power system calculations the equivalent circuits of three-phase transformers are drawn as one phase of the equivalent star network. The ratio a_{PS} (for example) of a three-phase transformer is the unloaded voltage ratio. Whether this is the true turns ratio depends on the winding connections. Also, an inherent phase displacement may be involved. Thus we can regard the ratio a_{PS} as a complex number.

Example

Consider a 33 / 11 kV Yd1 transformer. The HV winding is nominated as the primary. The voltage ratio $=33/11=3$, but the turns ratio of the windings is $\frac{33/\sqrt{3}}{11}=\sqrt{3}$. The clock hour figure = 1, therefore the LV lags the HV by 30°, or HV leads the LV by 30°. Hence the complex ratio $a_{HL}=3\angle 30°$. The positive sequence diagram is drawn in terms of per-unit quantities as follows:



$$Z_1 = Z_L = R_L + jX_L$$
$$= \text{the leakage impedance}$$

Figure 3.9 – Positive sequence equivalent circuit of example transformer

The subscripts "*A*" and "*a*" refer to the HV and LV windings ("*a*" phase). These could be replaced by "*A*(1)" and "*a*(1)" respectively to emphasize that the circuit is valid only for positive phase sequence.

Three-Phase Transformers – Negative Sequence Equivalent Circuits

The negative sequence impedance of a transformer, because it is a passive component, always equals the positive sequence impedance, which is the leakage impedance of the windings. While the impedance is independent of the direction of phase rotation, reversing the phase rotation reverses the sign of the phase angle in the complex ratio. Therefore the negative sequence equivalent circuit is obtained by reversing the sign of any phase shifters in the positive sequence equivalent circuit. For the transformer in the previous example, this illustrated in the following diagram:

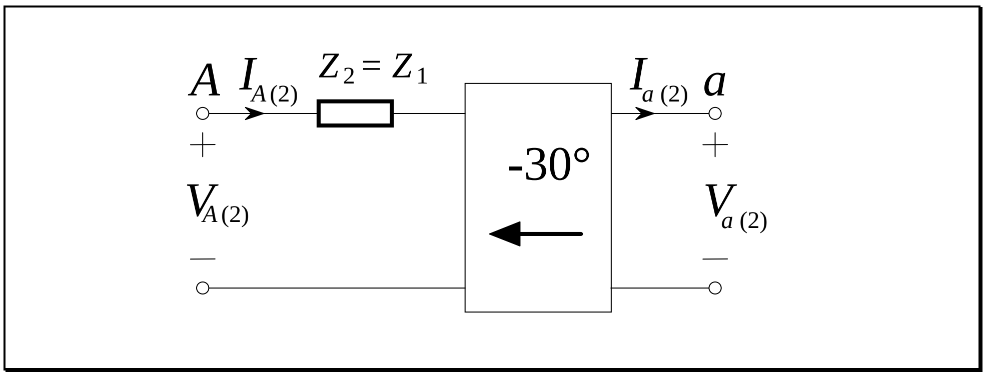


Figure 3.10 – Negative sequence equivalent circuit of example transformer

Three-Phase Transformers – Zero Sequence Equivalent Circuits

Of all the standard core constructions the three-limb core is an exception in that it does not provide a closed iron path for the zero sequence flux. This fact introduces complications which we will ignore at first, and consider in a later section.

A fundamental difference between zero sequence and positive sequence performance of a transformer is the inherent phase displacement. With the usual standard connections, which have positive sequence phase displacements of -30°, 0°, or +30°, the corresponding zero sequence phase displacement is always zero, and no phase shifters appear in the equivalent circuit.

A second difference is that the configuration of the zero sequence path(s) through the transformer may be different to the positive sequence path(s). The zero sequence paths depend on connections to each winding. We can construct a partial equivalent circuit for each type of winding connection, then put them together to match the configuration of the particular transformer.

(a) Star Winding without Neutral Connection

If the neutral terminal is not used, then there is no path for the zero sequence currents, and the winding must be open-circuited in the zero sequence equivalent circuit:

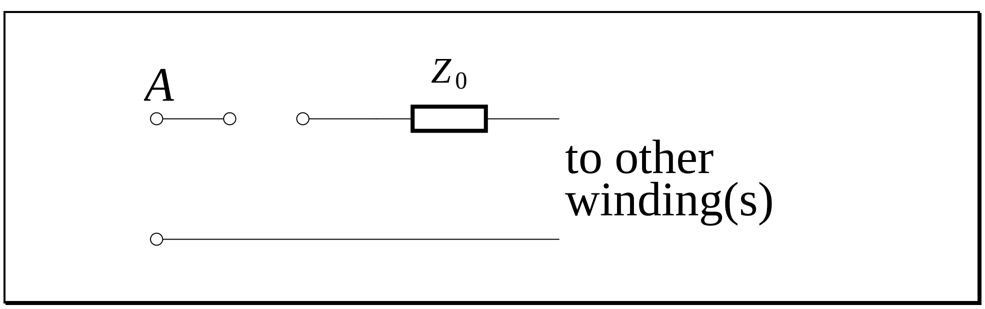


Figure 3.11

(b) Star Winding with Neutral Earthed Directly

Normal transformer action operates, and $Z_0 = Z_1$ = leakage impedance.

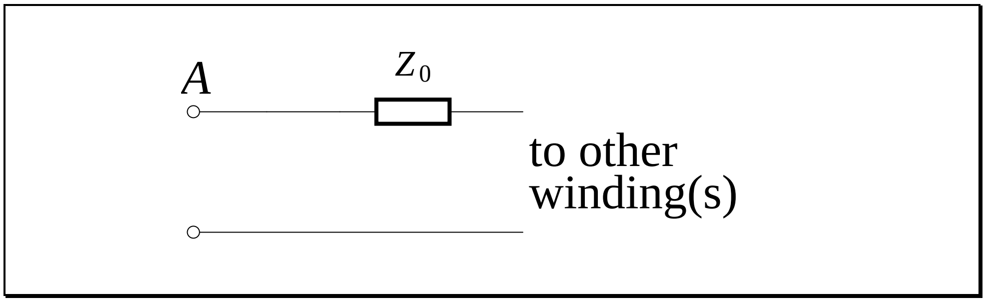


Figure 3.12

(c) Star Winding with Neutral Earthed via an Impedance

Z_N = neutral earthing impedance and $Z_0 = Z_1$ = leakage impedance.

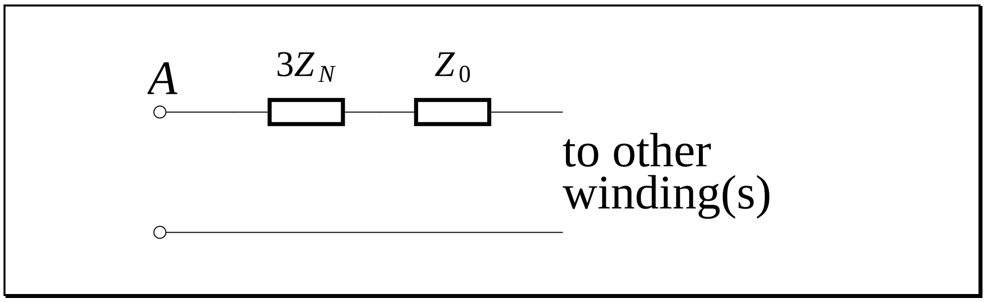


Figure 3.13

(d) Delta Winding

There is a closed zero sequence loop around the delta, but no zero sequence current can enter or leave via the line terminals. Thus the winding is open-circuited as seen from the line terminals, but short-circuited internally. $Z_0 = Z_1$ = leakage impedance.

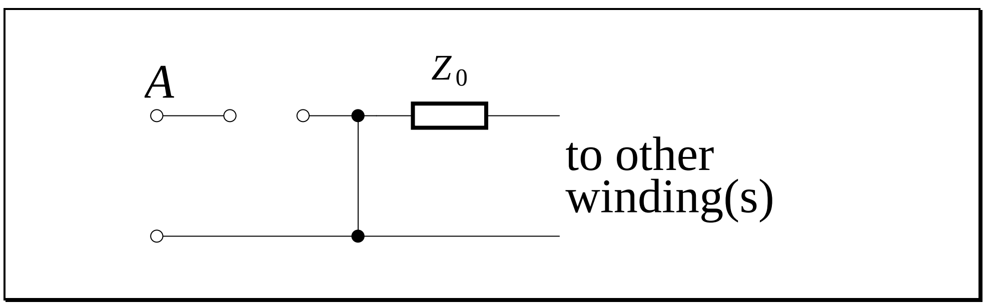


Figure 3.14

(e) Zig-Zag Winding with Neutral Earthed Directly

The zig-zag connection is shown below:

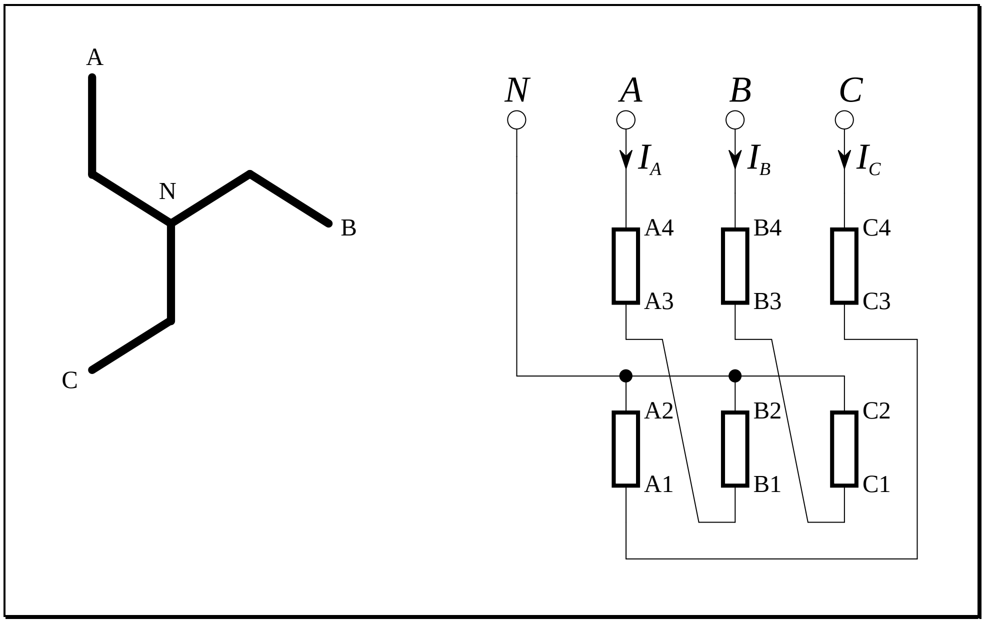


Figure 3.15 – Zig-Zag Winding Connection

Windings A1-A2 and A3-A4, $N/2$ turns each, are wound on the "A" limb of the magnetic circuit. Other limbs are wound similarly. Then the mmf produced by each winding is:

$$\begin{aligned}\mathscr{F}_A &= \frac{N}{2}\left(I_A - I_C\right)\\ \mathscr{F}_B &= \frac{N}{2}\left(I_B - I_A\right)\\ \mathscr{F}_C &= \frac{N}{2}\left(I_C - I_B\right)\end{aligned} \tag{3.17}$$

For positive sequence currents:

$$\mathscr{F}_A = \frac{NI_A}{2}(1-h) = \frac{\sqrt{3}}{2} NI_A \angle -30°$$
$$\mathscr{F}_B = \frac{NI_A}{2}(h^2-1) = \frac{\sqrt{3}}{2} NI_A \angle -150°$$
$$\mathscr{F}_C = \frac{NI_A}{2}(h-h^2) = \frac{\sqrt{3}}{2} NI_A \angle -270° \tag{3.18}$$

The core mmfs lag the line currents by 30°, which is the same as for a delta winding.

For zero sequence currents:

$$\mathscr{F}_A = \mathscr{F}_B = \mathscr{F}_C = 0 \tag{3.19}$$

There is no zero sequence mmf, hence no zero sequence flux, and no coupling to other windings. The zero sequence impedance Z_0 in this case is the leakage impedance between the sub-windings A1-A2 and A3-A4.

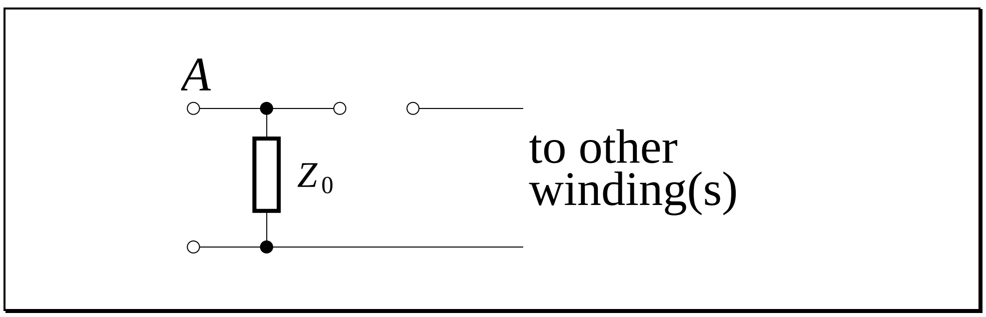
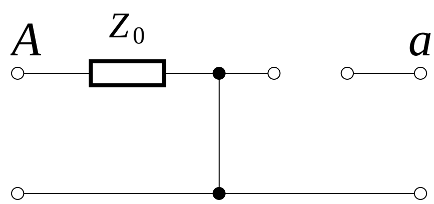


Figure 3.16

Example

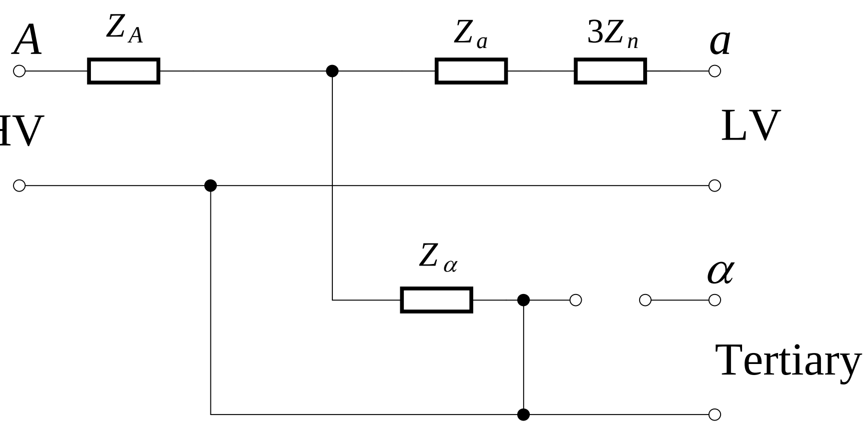
Zero sequence network for a HV star to LV delta transformer. HV neutral solidly earthed.



$Z_0 = Z_1 =$ leakage impedance

Example

Zero sequence network for a star-star transformer with delta tertiary winding. HV neutral solidly earthed, LV neutral earthed via impedance Z_n.



Z_A, Z_a and Z_α are leakage impedances identical to those in the positive sequence circuit.

Effect of Three-Limb Core on the Zero Sequence Equivalent Circuits

The three-limb core does not provide a closed path for the zero sequence flux, which has to return via a high reluctance path outside the steel core:

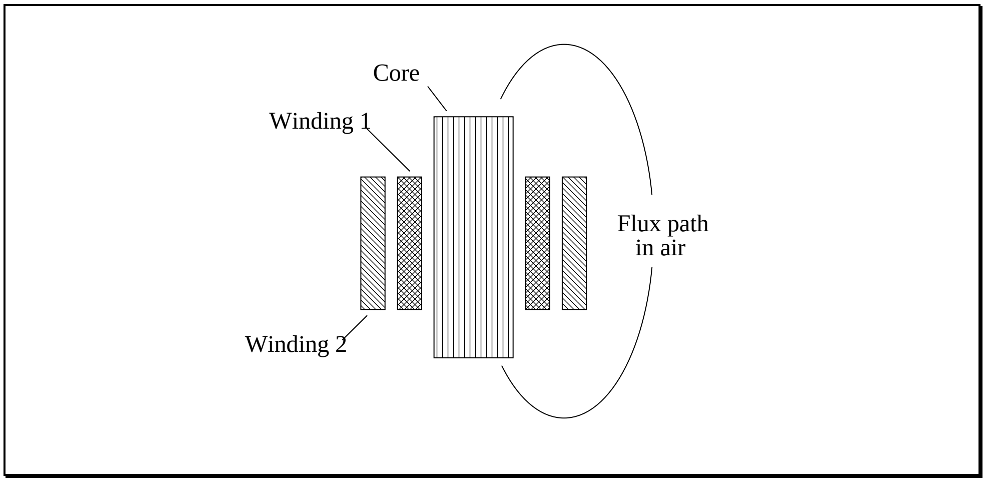


Figure 3.17 – Flux path for a Three-limb Transformer

The magnetic equivalent circuit is:

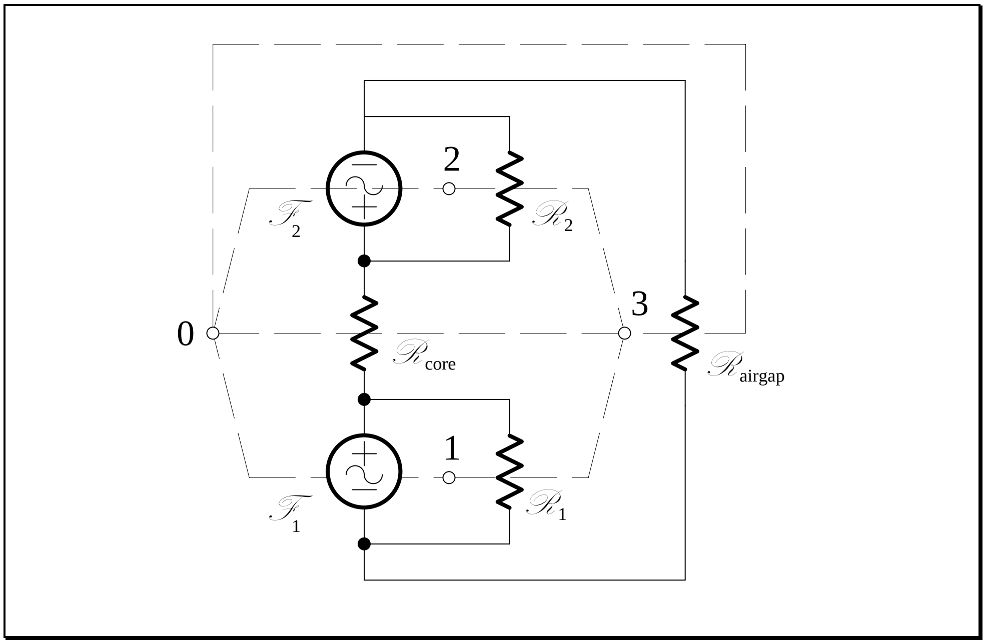


Figure 3.18 – Magnetic equivalent circuit for a Three-limb Transformer

The equivalent electric circuit can be derived directly from the magnetic circuit by the topological principle of duality. This topological technique is demonstrated in Figure 3.18. A node is marked within each mesh of the magnetic circuit, and a reference node is marked outside the circuit. These nodes are then joined by branches, one of which passes through each element of the magnetic circuit. For each reluctance in a mesh of the magnetic circuit, there is an inductance connected to the corresponding node of the electric circuit. Where a reluctance is common to two meshes in the magnetic circuit, the corresponding inductance connects the corresponding nodes of the electric circuit. For each magnetomotive force there is a corresponding emf between nodes.

The electric equivalent circuit is therefore:

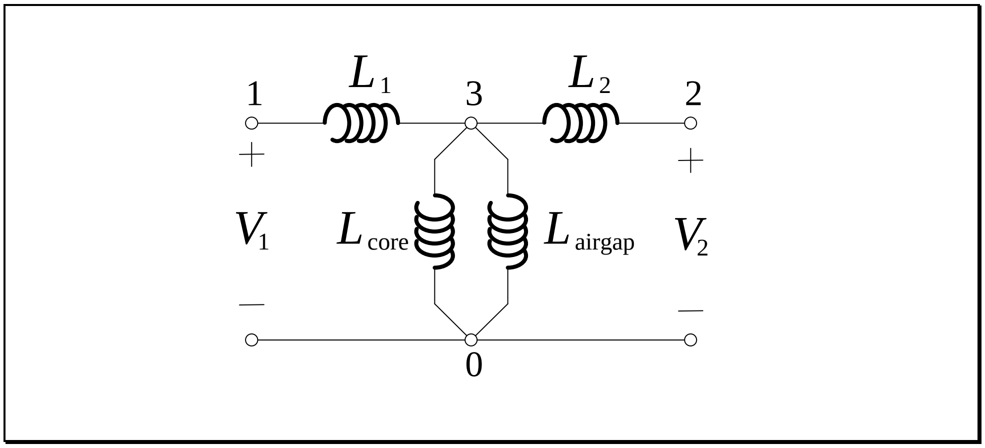


Figure 3.19 – Equivalent circuit of a three-winding power transformer

where:

$$
\begin{aligned}
V_1 &= \text{primary voltage} \\
V_2 &= \text{secondary voltage} \\
L_1 &= \text{primary leakage inductance} \\
L_2 &= \text{secondary leakage inductance} \\
L_{\text{core}} &= \text{core magnetising inductance} \\
L_{\text{airgap}} &= \text{airgap magnetising inductance}
\end{aligned}
\tag{3.20}
$$

The large effective airgap makes the magnetising impedance much lower, and much more linear, than would be the case without the airgap. Actually L_{core} has little effect, and L_{airgap} dominates the magnetising impedance.

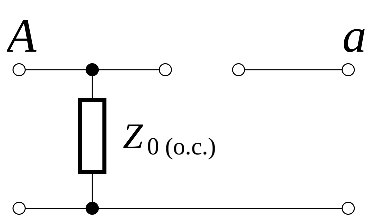
The low magnetising impedance has the same effect as a delta tertiary winding (of relatively high impedance) by providing an additional path to the zero sequence currents. The effect is further accentuated when the transformer with the three-limb core is placed in a metal tank. The tank forms a short-circuited turn around a portion of the zero sequence flux, acting as another weakly coupled delta winding.

As shown in Figure 3.19, the zero sequence performance of the three-limb core type transformer can not be adequately described by a single impedance, but requires three impedances in a T (or equivalent Π) network.

In practice, the impedances required to construct the T network may not be known, and in any case, the calculation is inaccurate because of nonlinearities. Reasonable results can generally be obtained by ignoring the effect of the three-limb core. A notable exception is the case of a star-star transformer with one neutral floating.

Example

Zero sequence network for a HV star to LV star transformer on a three-limb core. HV neutral solidly earthed, LV neutral not connected.



There is no zero sequence path through the LV winding. The zero sequence open-circuit impedance is seen from the HV side. $Z_{0(\text{o.c.})}$ is predominantly a magnetising impedance, significant for a three-limb core, but practically infinity for other types.

With three-winding transformers the effect of the three-limb core gets more complicated. With reference to the previous three-winding example, one might be tempted to add a magnetising impedance to the junction of the three partial leakage impedances Z_A, Z_a and Z_α, but this does no give a valid equivalent circuit, which would require at least six impedances.

Ability to Supply Unbalanced Loads

(a) Single-phase line-to-line load

Assume a load between lines "b" and "c". We can solve the problem by treating the load as a line-to-line fault, and use the method of sequence networks to find the short-circuit current I_{SC}:

$$I_{SC} = -j\frac{\sqrt{3}E_1}{Z_1 + Z_2} = -j\frac{\sqrt{3}E_1}{2Z_1} \qquad (Z_2 = Z_1) \tag{3.21}$$

The open-circuit voltage is:

$$V_{OC} = -j\sqrt{3}E_1 \tag{3.22}$$

Hence, the source impedance (by Thévenin's Theorem) is:

$$Z_{\text{source}} = \frac{V_{OC}}{I_{SC}} = 2Z_1 \tag{3.23}$$

Since Z_1 is always small the transformer has no difficulty in supplying the single-phase line-to-line load.

(b) Single-phase line-to-neutral load

Assume the load is on phase "a". We can solve the problem by treating the load as a line-to-earth fault, and the method of sequence networks to find the short-circuit current I_{SC}:

$$I_{SC} = \frac{3E_1}{Z_0 + Z_1 + Z_2} = \frac{3E_1}{Z_0 + 2Z_1} \tag{3.24}$$

The open-circuit voltage is:

$$V_{OC} = E_1 \tag{3.25}$$

Hence, the source impedance (by Thévenin's Theorem) is:

$$Z_{\text{source}} = \frac{V_{OC}}{I_{SC}} = \frac{Z_0 + 2Z_1}{3} \tag{3.26}$$

A problem arises if Z_0 is large, e.g. if the neutral of a star-star transformer is not earthed. Not only does the voltage then collapse on the loaded phase because of excessive source impedance, but overvoltages are produced on the other two phases. In the extreme case (infinite Z_0) the magnitude of the voltages on the unloaded phases becomes $\sqrt{3}$ p.u.

Summary

- The different core constructions of three-phase transformers lead to different electric and magnetic equivalent circuits, and therefore transformer behaviour.

- The effect of the winding connections of a transformer also affects its behaviour.

- The positive and negative sequence equivalent electric circuits of a transformer consist entirely of leakage impedance, but may involve a phase shift (depending on winding connections).

- The zero sequence equivalent electric circuits for a transformer are highly dependent on the winding connections.

- The three-limb core transformer has a T-equivalent zero sequence equivalent electric circuit, but in practice we ignore the effect of the three-limb core (unless the neutral of a star-star transformer is left floating).

- Transformers generally have no difficulty supplying single-phase line-to-line loads.

- Transformers with large zero sequence impedances have difficulty supplying single-phase line-to-neutral loads – the voltage tends to collapse on the loaded phase and overvoltages occur on the other two.

References

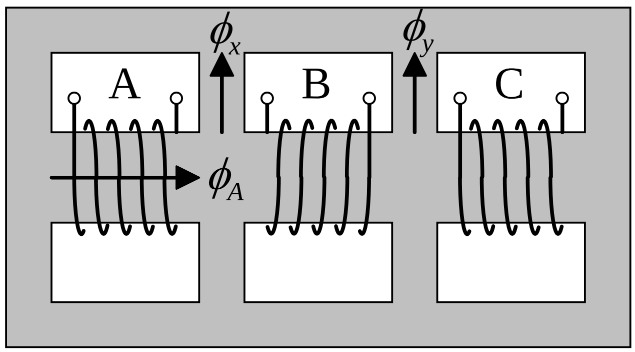
Carmo, J.: *Power Circuit Theory Notes*, UTS, 1994.

Truupold, E.: *Power Circuit Theory Notes*, UTS, 1993.

Exercises

1.

For the shell type transformer shown, determine the magnitude of fluxes ϕ_x and ϕ_y given that symmetrical 3-phase voltages are applied to the windings, taking the main flux ϕ_a to be 1 p.u.



(a) With winding polarities shown.

(b) With B phase wound in the same direction as A and C.

2.

A single-phase three-winding transformer has the following ratings:

HV	10 kV, 400 kVA
LV	600 V, 400 kVA
TV	1.2 kV, 100 kVA

The transformer is fed from the LV side at 600 V. The TV winding is loaded with a $15\,\Omega$ resistor, and the HV winding is connected to a capacitive reactance of $260\,\Omega$. Use ideal transformer modelling to determine:

(a) All three winding currents.

(b) kVA loading of each winding.

3.

A 50 kVA single-phase 800 V / 200 V transformer has leakage impedance of $(1.2 + j2.0)\%$. It is reconnected as a 800 V / 1000 V autotransformer.

(a) What is the new kVA rating?

(b) What is the leakage impedance based on the new ratings?

4.

A single-phase 75 kVA transformer has three windings 1, 2, & 3, rated at 2400, 600 & 240 V respectively. Short-circuit test gave the following results:

(i) Winding 2 shorted, winding 3 open:

$I_1 = 31.25\text{ A}$, $V_1 = 120\text{ V}$, $P_1 = 750\text{ W}$

(ii) Winding 3 shorted, winding 2 open:

$I_1 = 31.25\text{ A}$, $V_1 = 135\text{ V}$, $P_1 = 810\text{ W}$

(iii) Winding 3 shorted, winding 1 open:

$I_2 = 125.0\text{ A}$, $V_2 = 30\text{ V}$, $P_1 = 815\text{ W}$

Determine the constants, expressed as percent values, of the equivalent circuit for this transformer. Neglect the excitation current and corrections for temperature.

5.

A single-phase transformer is rated 60 Hz, 100 kVA, 13200 V / 200 V, and at those ratings has an impedance of $(0.39 + j6.12)\%$.

(a) What would happen if a second identical transformer were to be connected in parallel, but by mistake one of its windings had reverse polarity? Assume the 60 Hz HV supply maintains a constant voltage at 13.2 kV.

(b) What would happen if the transformer were to be used at rated voltage in a 50 Hz system?

(c) The transformer is to be connected to a 50 Hz 11 kV network.

(i) What kVA rating would you now assign to the transformer?

(ii) What is the percent impedance at the revised rating?

6.

The rating plate of a 375 MVA transformer gives the following information:

Winding	Rated kV	Rated MVA	Impedances on 375 MVA base on principal tapping	
HV	330	375	HV-LV	16.47%
LV	132	375	HV-TV	47.37%
TV	11	5	LV-TV	29.64%

Connection symbol: YN,a0,d1

a) Sketch a circuit diagram, ignoring facilities for tap changing, but showing main winding connections and polarities.

b) Calculate the complex ratios: HV / LV, HV / TV, LV / TV.

c) Sketch the positive sequence equivalent circuit, and calculate its component impedances. Assume all resistances and the no-load current are negligible.

7.

The transformer in Q6 delivers 100 MW 0 Mvar to the 132 kV bus at nominal voltage. The 11 kV winding is connected to a capacitor bank rated at 5 Mvar, 11 kV. Calculate the magnitude and phase of all line and phase currents, taking the 'a' phase of the 132 kV bus as the reference. Also calculate the megawatts and megavars taken from the 330 kV bus.

8.

For the transformer in Q6:

(a) Sketch the negative sequence equivalent circuit.

(b) Sketch the zero sequence equivalent circuit, assuming the neutral to be solidly earthed.

9.

A three-phase 500 kVA 33 kV / 500 V transformer has an impedance of $(0.39 + j6.12)\%$. Calculate:

(a) The maximum kVA the transformer can supply to a single phase line-to-line connected load at 500 V without exceeding the rated current.

(b) The regulation with a single phase load as in (a), when the load power factor is:

(i) 0.9 lagging
(ii) 0.9 leading